

Some observations about regular maps

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A map on an orientable surface is called ‘orientably-regular’ if its automorphism group has a single orbit on arcs (incident vertex-edge pairs), and is then called ‘reflexible’ or ‘chiral’ depending on whether or not it admits reflections (for example, fixing an arc but swapping the two faces incident with it). In every such map with a high degree of symmetry, all vertices have the same valency, say k , and all faces have the same size, say m , and then the (ordered) pair $\{m, k\}$ is called the ‘type’ of the map.

Writing a book with Gareth Jones, Jozef Širáň and Tom Tucker) on regular maps has prompted us to review and extend what is known about them. Various questions have arisen naturally, including what I believe is the most important unanswered one, namely whether chirality is more prevalent than reflexivity. Other questions include these: What types $\{m, k\}$ occur the most frequently among orientably-regular maps on hyperbolic surfaces? What kinds of groups are the most prevalent as the group of orientation-preserving automorphisms? (Simple groups? insoluble groups? soluble groups? 2-groups?) Is chirality more prevalent than reflexivity for a given type? I will give some partial answers to these questions, with reference to computational evidence. These answers may be surprising.