

Subset products and derangements

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In the past two decades there has been intense interest in products of subsets in finite groups. Two important examples are Gowers' theory of Quasi Random Groups and its applications by Nikolov, Pyber, Babai and others, and the theory of Approximate Groups and the Product Theorem of Breuillard-Green-Tao and Pyber-Szabo on growth in finite simple groups of Lie type of bounded rank, extending Helfgott's work.

These deep theories yield strong results on products of three subsets (covering, growth). What can be said about products of two subsets? I will discuss a recent joint work with Michael Larsen and Pham Tiep on this challenging problem, focusing on products of two normal subsets of finite simple groups, and deriving a number of applications.

Our main application concerns derangements (namely, fixed-point-free permutations), studied since the days of Jordan. We show that every element of a sufficiently large finite simple transitive permutation group is a product of two derangements. Related results and problems will also be discussed.

The proofs combine group theory, algebraic geometry and representation theory; it applies the proof by Fulman and Guralnick of the Boston-Shalev conjecture on the proportion of derangements.