Higher Kazhdan projections and the Baum-Connes conjecture

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The Baum-Connes conjecture, if it holds for a certain group, provides topological tools to compute the K-theory of its reduced group C*-algebra. This conjecture has been confirmed for large classes of groups, such as amenable groups, but also for some Kazhdan’s property (T) groups. Property (T) and its strengthening are driving forces in the search for potential counterexamples to the conjecture. Having property (T) for a group is characterised by the existence of a certain projection in the universal group C*-algebra of the group, known as the Kazhdan projection. It is this projection and its analogues in other completions of the group ring, which obstruct known methods of proof for the Baum-Connes conjecture. In this talk, I will introduce a generalisation of Kazhdan projections. Employing these projections we provide a link between surjectivity of the Baum-Connes map and the $\ell^2$-Betti numbers of the group. A similar relation can be obtained in the context of the coarse Baum-Connes conjecture. This is based on joint work with Kang Li and Piotr Nowak.