PROBABILITY

Generation of first passage times for diffusion processes: an overview of simulation techniques

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Many biological or physical applications require to simulate random variables with a given probability distribution. The aim of our study is to focus on a particular random variable: the first passage time (FPT) of a diffusion process. We introduce \((X_t)\) the unique solution of the following SDE:

\[
dX_t = b(X_t) \, dt + \sigma(X_t) \, dB_t, \quad X_0 = x,
\]

where \((B_t)\) stands for a one-dimensional Brownian motion and define \(\tau_L\) the first passage time through the level \(L\). We propose an overview of several simulation techniques.

- The classical way is to use efficient algorithms for the simulation of sample paths, like discretization schemes. Such methods permit to obtain approximations of the first-passage times as a by-product.

- Another approach based on a random walk on spheroids permit in particular cases to express the first passage time as the limit of a random walk. It suffices therefore to describe precisely the convergence of this stochastic process and to introduce a stopping procedure.

- Finally we present a new rejection sampling algorithm which permits to perform an exact simulation of the first-passage time for general one-dimensional diffusion processes. The main ideas are based both on a previous algorithm pointed out by A. Beskos et G. O. Roberts which uses Girsanov's transformation and on properties of Bessel paths.

References
