Maximal regularity is a useful tool for solving abstract parabolic evolution equations. A variant of the one-phase quasistationary Stefan problem that we consider can be reduced to a single evolution equation. We tackle this problem using $H^\infty$-calculus to show that the operator in the evolution equation has maximal regularity and then apply an existence theorem for this type of evolution equation. We use additionally an assumption that one particular result on the solvability of a degenerate oblique derivative problem extends in an appropriate way.