Limit states of multi-component discrete dynamical systems

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Object. We study models of multicomponent discrete dynamic conflict systems with attractive interaction, which are characterized by a positive value that is called the attractor index. Consider the set of discrete probability measures \( \mu_i \in M_1^+(\Omega) \) on finite space \( \Omega = \{\omega_1, \ldots, \omega_n\} \), \( i = \overline{1, m} \). Each of these measures \( \mu_i \) can be identified with a stochastic vector \( p_i = (p_{ij})_{j=1}^n \), where

\[
p_{ij} = \mu_i(\omega_j), \quad i = \overline{1, m}, \quad j = \overline{1, n}.
\]

Consider the mapping \( \ast \)

\[
\{P_1^t, P_2^t, \ldots, P_m^t\} \rightarrow \{P_1^{t+1}, P_2^{t+1}, \ldots, P_m^{t+1}\}, \quad (1)
\]

which generates multi-component discrete dynamical systems with trajectories (1), where the coordinates of each vector \( P_i^t = (p_{ij})_{j=1}^n \) are changed according to equations

\[
p_{ij}^{t+1} = \frac{1}{z^t}(p_{ij}^t(\theta^t + 1) + \tau_j^t), \quad t = 0, 1, \ldots (2)
\]

Here \( \theta^t = \theta(P_1^t, P_2^t, \ldots, P_m^t) \) is a finite positive function, \( T^t = (\tau_j^t)_{j=1}^n \) is a vector with non-negative coordinates (attractor index), and \( z^t = \theta^t + 1 + W^t \) is normalizing denominator, \( W^t = \sum_{j=1}^n \tau_j^t \).

Main results. Theorem 1. Let all coordinates of vector \( w^t = (w_j^t)_{j=1}^n \), \( w_j^t := \frac{\tau_j^t}{W^t} \) be bounded and monotonic (increase or decrease independently one to other). Then for all \( i = \overline{1, m} \) there exist

\[
p_i^\infty = \lim_{t \to \infty} p_i^t
\]

and all limit vectors \( p_i^\infty \) coincide with the vector \( w^\infty \), i.e.

\[
p_{ij}^\infty = \frac{\tau_j^\infty}{W^\infty} \quad \forall j.
\]
Let us consider the different variants of attractor index $T^t$:

$$
\tau_j^t := \tau_{j,\min}^t = \min_i \{p_{ij}^t\}, \quad (3)
$$

$$
\tau_j^t := \tau_{j,\max}^t = \max_i \{p_{ij}^t\}, \quad (4)
$$

$$
\tau_j^t := \tau_j^t = \frac{1}{m} \sum_{i=1}^n p_{ij}^t, \quad (5)
$$

$$
\tau_{j_1}^t = \tau_{j_2}^t > 0, \quad j_1, j_2 = 1, n. \quad (6)
$$

**Theorem 2.** Let coordinates of the attractor index $T^t$ be given by one of the equations (3), (4), (5), (6). Then each trajectory of a dynamic system (1) with an initial state $\{p_1, p_2, \ldots, p_m\}$ converges to the fixed point $\{p_1^\infty, p_2^\infty, \ldots, p_m^\infty\}$

$$
p_i^\infty = \lim_{t \to \infty} p_i^t, \quad \forall \ i = 1, m,
$$

where the coordinates of $p_i^\infty$ have a view:

$$
p_{ij}^\infty = \frac{\tau_j}{W} \quad \forall \ i = 1, m, \quad j = 1, n. \quad (7)
$$

**Stability.** The limit state is unstable in cases (3)-(5), however it is stable only in the case (6), when all limit coordinates are equal to $\frac{1}{n}$.

**Application.** Such model of dynamic systems can describe the dynamics of real processes. Attractor index can describe a real external influence on a certain system (for example, information influence on a society). System behavior can be controlled or described by setting attractor index which can be exposed to such influence.

**REFERENCES**

