On Cayley isomorphism property for abelian groups

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A finite group $G$ is called a DCI-group if every two isomorphic Cayley digraphs over $G$ are Cayley isomorphic, i.e. there exists an isomorphism between these digraphs that is also an automorphism of $G$. One of the motivations to study DCI-groups comes from the Cayley graph isomorphism problem. Suppose that $G$ is a DCI-group. Then to determine whether two Cayley digraphs $\text{Cay}(G,S)$ and $\text{Cay}(G,T)$ are isomorphic, we only need to check the existence of $\varphi \in \text{Aut}(G)$ with $S^{\varphi} = T$. The latter, usually, is much easier.

The definition of a DCI-group goes back to Ádám who conjectured [1], in our terms, that every cyclic group is DCI. This conjecture was proved to be false. The problem of determining all finite DCI-groups was raised by Babai and Frankl [2]. One of the crucial steps towards the classification of all DCI-groups is to determine abelian DCI-groups. It was proved that every abelian DCI-group is the direct product of groups of coprime orders each of which is elementary abelian or isomorphic to $\mathbb{Z}_4$ (see [3, Theorem 8.8]). However, the classification of abelian DCI-groups is far from complete. In the talk we discuss on new infinite families of abelian DCI-groups and approaches to determining whether a given group is DCI.

References

