

Semigroups for flows on limits of graphs

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Transport of goods is nowadays of extreme importance and indispensable considering what mankind needs for daily life. Now imagine a start-up company shipping special goods all over the world. Of course, the company starts with a small network of customers. However, assuming the company grows and retains the already existing routes and customers, the ship network grows and grows. It might come to the point in the development of the company, that one actually lost the view on all specific routes but only knows how the network works since it becomes too big. However, one still wants to know how the transport is going on the whole network.

That a network is growing, through adding vertices and edges, means that one has a sequence of graphs and each graph is a subgraph of the subsequent graph in the sequence, describing the above mentioned situation of growing networks. We will describe the transition from finite to infinite graphs by means of direct limits in a certain category. The approximation of the transport process on the direct limit graph, is done by a version of the (first) Trotter–Kato approximation theorem, which is originally due to T. Kato [2, Chapter IX, Thm. 3.6] and H.F. Trotter [3, Thm. 5.2 & 5.3] and modified by a version by K. Ito and F. Kappel [1, Thm. 2.1]. We will also present an extension the work of Ito and Kappel which is related to the well-known second Trotter–Kato theorem.

References

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- [3] H. F. Trotter. Approximation of semi-groups of operators. *Pacific J. Math.*, 8:887–919, 1958.