Topological estimates of the number of vertices of minimal triangulation

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From the beginning of the algebraic topology, then also called the combinatorial topology, i.e. from the beginning of the 20th century, its basic object is the simplicial complex. The representation of a given topological space $X$ as a simplicial complex (i.e. a homeomorphism with it) is called triangulation. The name comes from the fact that when space $X$ is a two-dimensional surface, triangulation means representing it as the union of adjacent triangles with edges meeting at the vertices. In the case of higher dimensions, the basic cells are the "$i$" - dimensional simplices. One of the natural questions is to find a triangulation with the minimum number of vertices, respectively of all simplexes (or estimate these numbers). This lecture will be devoted to this problem. We will present a new method based on the notion of covering type estimating from below the number of vertices by the weighted length of the elements in the cohomology ring $H^*(X)$, or the weighted Lusternik-Schirelmann category theory. As a consequence, we got not only a unified method of proof of estimates of the number of vertices of the minimal triangulations derived originally by ad hoc combinatorial methods, but also sharper estimates, or estimates for the families of manifolds not studied earlier.

References

