Sequences in $\mathbb{Z}_n$ with Distinct Partial Sums

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Let $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ be the integers modulo n. For a sequence of elements $a_1, \ldots, a_k$ of $\mathbb{Z}_n$, define its *partial sums* $b_0, \ldots, b_k$ by $b_0 = 0$ and $b_i = a_1 + \cdots + a_i$ for $1 \leq i \leq k$. For which subsets $S \subseteq \mathbb{Z}_n \setminus \{0\}$ is it possible to order the elements of S so that the partial sums are distinct?

When the sum of the elements of S is 0, there can be no such ordering. Alspach conjectures that this is the only obstacle; that is, every subset S whose sum is nonzero has an ordering with distinct partial sums.

We show how to translate the problem into one of finding monomials with non-zero coefficients in particular polynomials over $\mathbb{Z}_p$, where p is a prime divisor of n, using Alon's Combinatorial Nullstellensatz. The approach offers hope for a full proof of the conjecture, and can be used in conjunction with a computational approach in cases where $n = pt$ with p prime and t and $|S|$ small.