Null scrolls, B-scrolls and associated evolute sets in Lorentz-Minkowski 3-space

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In classical differential geometry in Euclidean space, the Bonnet's theorem states that there are two surfaces of constant mean curvature parallel a surface of constant positive Gaussian curvature. These two constant mean curvature surfaces are so-called harmonic evolutes of each other. In this short presentation, we present results of the analogous investigation in Lorentz-Minkowski 3-space, however, restricted to the case of surfaces that have no Euclidean counterpart, the quasi-umbilical surfaces, [1]–[4]. These surfaces are characterized by the property that their shape operator is not diagonalizable, and they can be parametrized as null scrolls or B-scrolls, [6]. In [5] we have shown that they are the only surfaces whose evolute set degenerates to a curve. The curve is of either null or spacelike causal character, and we analyse them respectively.

References

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