The Golden ratio and high dimensional mean inequalities

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The classical inequalities between means state that

$$\min\{a,b\} \leq \left(\frac{a^{-1}+b^{-1}}{2}\right)^{-1} \leq \sqrt{ab} \leq \frac{a+b}{2} \leq \max\{a,b\}, \tag{1}$$

for any $a, b > 0$, with equality if and only if $a = b$.

One can naturally extend (1) considering means of n-dimensional compact, convex sets. Notice that means of K and $-K$ are commonly known as symmetrizations of K. In this context, we will show that in even dimensions, if K has a large Minkowski asymmetry, then the corresponding inequalities between the symmetrizations of K can no longer be optimal. Especially in the planar case, we compute that the range of asymmetries of K for which the inequalities between the symmetrizations of K can be optimal is $[1, \phi]$, where ϕ is the Golden ratio. Indeed, we introduce the Golden House, (up to linear transformations) the only Minkowski centered set with asymmetry ϕ such that the symmetrizations of K are successively optimally contained in each other.