Real Cayley-Dickson algebras: doubly alternative elements and zero divisor graphs

Svetlana Zhilina

Lomonosov Moscow State University

s.a.zhilina@gmail.com

Due to the non-alternativity of Cayley-Dickson algebras with dimension at least 16, there appear zero divisors which are hard to study and to classify, except for some particular cases. At present most of the authors restrict their attention to the algebras of the main sequence where all parameters which determine the Cayley-Dickson construction are assumed to be equal to $-1$. We mention in particular the works by Moreno and Biss, Dugger, and Isaksen. Moreno’s key idea was to study doubly alternative zero divisors, that is, such elements that their components are both alternative elements of the previous algebra. This notion was then extended to the Cayley-Dickson split-algebras and led to similar results.

The talk is devoted to zero divisors in arbitrary real Cayley-Dickson algebras whose components satisfy some additional conditions on the norm and alternativity. We are interested in the patterns which they form in orthogonality and zero divisor graphs, and these patterns appear to be hexagonal. In the case of the algebras of the main sequence, these hexagons can be extended to the so-called double hexagons. Moreover, the vertices of a double hexagon have a convenient multiplication table which has a block structure.