Approximation of the two variables Discontinuous Functions by Discontinuous Interlination Splines using Triangular Elements

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The problem of approximation of smooth functions by continuous construction with sufficient completeness was considered in the work of many researchers. However, as a rule, all these problems are only pleasant exceptions or the result of excessive idealization. More often, the objects of study are mathematically described by functions with discontinuities, kinks, and other violations of smoothness. Such nonsmooth functions include profile shock waves generated by powerful acoustic emitters, or the shape of dunes in the desert, which provides characteristic sharpening. Discontinued objects also very often arise in tasks using remote methods. Thus, the detection of cracks in industrial products using non-destructive testing is an important task in flaw detection, as is the determination of deviations of the internal structure of the product from the standard. In many tasks of geophysics, the determination of the location of bound aries separating blocks with different physical properties is the first step in further studies aimed at determining the physical quantities characterizing the internal structure of the Earth. In computed tomography, when studying the internal structure of a body, it is useful to take into account its heterogeneity, that is, different densities in different parts of the body. In general, remote methods are one of the most promising areas of environmental studies. They serve as the most important source of objective and operational information in various phenomena occurring in the geographic shell of the Earth, and are an effective tool for monitoring the state of the environment and solving environmental management problems. It is obvious that further progress in the development of natural sciences is inextricably linked with the use of remote research methods. This work belongs to a series of works by the authors aimed at the study and improvement of mathematical models in computed tomography. To date, tomography has developed many computational methods, algorithms and software tools aimed at restoring the internal properties of an object. They perform well when restoring objects with smooth properties, but give unsatisfactory results for objects with discontinuous characteristics. Therefore, there is a need to create mathematical methods for approximating discontinuous functions for a more accurate idea of the structure of the studied object. The mathematical foundations of tomography were laid at the beginning of
the last century in the works of the German scientist J. Radon, who developed the theory of the transformation of functions of many variables (Radon transformation). According to these transformations, the function of many variables can be characterized not only by its values at points of multidimensional space, but also by integrals from this function taken over an infinite set of lines or planes. A series of works by authors [18-20] devoted to solving the flat problem of radon computed tomography using the heterogeneity of the internal structure of a two-dimensional body. For this purpose, it is advisable to use function interlination operators, since these operators restore (possibly approximated) functions on their known traces on a given system of lines. They provide an opportunity to construct operators whose integrals from these lines (linear integrals) will be equal to integrals from the most renewable function. That is, interlination is a mathematical apparatus, naturally related to the task of restoring the characteristics of objects according to their known projections. This article is a continuation of this article series. Paper is devoted to the development of a method for approximating two variables discontinuous functions by discontinuous interlination splines using arbitrary triangular elements. Experimental data are one-sided traces of a function along a system of given lines, such data are used in remote methods, in particular in tomography. The paper is also devoted to the development a method for approximating of two variables discontinuous functions by triangular elements that comprise one curved side. These methods make it possible to approximate the discontinuous function, using its more complex domains of definition and avoiding the Gibbs phenomenon.