

Nonlocal minimal graphs in the plane are generically sticky

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We discuss some recent boundary regularity results for nonlocal minimal surfaces in the plane. In particular, we show that nonlocal minimal graphs in the plane exhibit generically stickiness effects and boundary discontinuities. More precisely, if a nonlocal minimal graph in a slab is continuous up to the boundary, then arbitrarily small perturbations of the far-away data necessarily produce boundary discontinuities. Hence, either a nonlocal minimal graph is discontinuous at the boundary, or a small perturbation of the prescribed conditions produces boundary discontinuities. The proof relies on a sliding method combined with a fine boundary regularity analysis, based on a discontinuity/smoothness alternative. Namely, we establish that nonlocal minimal graphs are either discontinuous at the boundary or their derivative is Hölder continuous up to the boundary. In this spirit, we prove that the boundary regularity of nonlocal minimal graphs in the plane "jumps" from discontinuous to differentiable, with no intermediate possibilities allowed. In particular, we deduce that the nonlocal curvature equation is always satisfied up to the boundary. As a byproduct of our analysis, one describes the "switch" between the regime of continuous (and hence differentiable) nonlocal minimal graphs to that of discontinuous (and hence with differentiable inverse) ones. These results have been obtained in collaboration with Serena Dipierro and Ovidiu Savin.