

Quasibands and nonassociative, noncommutative lattices

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Nonassociative idempotent magmas arise naturally in various settings such as the faces of a building or chains in modular lattices. In this talk I will describe a variety of magmas we call *quasibands*, which arise as (sub)reducts of bands (idempotent semigroups): in a band (B, \cdot) , define a new operation \circ by $x \circ y = xyx$. This is analogous to how quandles arise as subreducts of groups under the conjugation operations. The quasiband operation \circ is sometimes used as a notational shorthand (especially in the theory of noncommutative lattices) or to characterize band properties. For instance, (B, \circ) is associative, hence a left regular band, if and only if (B, \cdot) is a regular band.

The variety of quasibands is defined by 4 identities. A main result is that this is precisely the variety of \circ -subreducts of bands. In addition, I will talk about the natural preorder and natural partial order on a quasiband, the center of a quasiband, the relationship between free quasibands and free bands, and some enumeration of quasibands for low orders.

In the case of a noncommutative lattice (B, \wedge, \vee) , the corresponding “double quasiband” (B, \wedge, \vee) can be viewed as a nonassociative, noncommutative lattice. I will discuss the relationship between the two structures for some of the more commonly studied classes of noncommutative lattices (quasilattices, paralattices, skew lattices, etc.)

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