

Kirby diagrams, edge-colored graphs and trisections of PL 4-manifolds

Maria Rita Casali

University of Modena and Reggio Emilia

casali@unimore.it

Paola Cristofori

University of Modena and Reggio Emilia

paola.cristofori@unimore.it

It is well-known that any *framed link* (L, c) uniquely represents the 3-manifold $M^3(L, c)$ obtained from S^3 by Dehn surgery along (L, c) , as well as the PL 4-manifold $M^4(L, c)$ obtained from \mathbb{D}^4 by adding 2-handles along (L, c) . Moreover, if trivial dotted components are also allowed (i.e. in case of a *Kirby diagram* $(L^{(*)}, c)$), the associated PL 4-manifold $M^4(L^{(*)}, c)$ is obtained from \mathbb{D}^4 by adding 1-handles along the dotted components and 2-handles along the framed components.

In the present talk we present the relationship between framed links and/or Kirby diagrams and the so called *crystallization theory*, which represents compact PL manifolds of arbitrary dimension by regular edge-colored graphs: in particular, we describe how to construct a 5-colored graph representing $M^4(L^{(*)}, c)$, directly “drawn over” a planar diagram of $(L^{(*)}, c)$.

As a consequence, the combinatorial properties of Kirby diagrams yield upper bounds for both the graph-defined invariants *gem-complexity* and *generalized regular genus* of the associated 4-manifold.

Further, the described relationship turns out to be strictly related to the possibility of studying *trisections* of 4-manifolds via edge-colored graphs, also in the extended case of compact PL 4-manifolds with connected boundary.