Decomposition formula for rough Volterra stochastic volatility models

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The research presented in this paper provides an alternative option pricing approach for a class of rough fractional stochastic volatility models. These models are increasingly popular between academics and practitioners due to their surprising consistency with financial markets. However, they bring several challenges alongside. Most noticeably, even simple nonlinear financial derivatives as vanilla European options are typically priced by means of Monte–Carlo (MC) simulations which are more computationally demanding than similar MC schemes for standard stochastic volatility models. In this paper, we provide a proof of the prediction law for general Gaussian Volterra processes. The prediction law is then utilized to obtain an adapted projection of the future squared volatility — a cornerstone of the proposed pricing approximation. Firstly, a decomposition formula for European option prices under general Volterra volatility models is introduced. Then we focus on particular models with rough fractional volatility and we derive an explicit semiclosed approximation formula. Numerical properties of the approximation for a popular model — the rBergomi model — are studied and we propose a hybrid calibration scheme which combines the approximation formula alongside MC simulations. This scheme can significantly speed up the calibration to financial markets as illustrated on a set of AAPL options.