On the boundary of the basins of attraction for the secant method applied to polynomials

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We investigate the discrete dynamical system $S$ defined on $\mathbb{R}^2$ given by the secant method applied to a real polynomial $p$. Every simple root $\alpha$ of $p$ has associated its basin of attraction $A(\alpha)$ formed by the set of points converging under $S$ towards $\alpha$ and $A^*(\alpha)$ its immediate basin of attraction.

We focus on the structure and dynamical behaviour of the boundary of the immediate basin of attraction of a root of $p$. We call *external* roots of $p$ the smallest and largest value and *internal* all the rest. If $\alpha$ is an internal root of $p$ then $\partial A^*(\alpha)$ is given by the stable manifold of a 4-cycle. Moreover we show that, under some hypothesis, those internal basins are simply connected.