Optimal control problem for a repulsive chemotaxis system

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The chemotaxis phenomenon can be understood as the directed movement of living organisms in response to chemical gradients. Keller and Segel [5] proposed a mathematical model that describes the chemotactic aggregation of cellular slime molds. These molds move preferentially towards relatively high concentrations of a chemical substance secreted by the amoebae themselves. Such mechanism is called *chemo-attraction* with production. However, when the regions of high chemical concentration generate a repulsive effect on the organisms, the phenomenon is called *chemo-repulsion*.

In this work, we want to study an optimal control problem for the (repulsive) Keller-Segel model and a bilinear control acting on the chemical equation in a 2D and 3D domains. The system can be written as:

\[
\begin{aligned}
\begin{cases}
\partial_t u - \Delta u - \nabla \cdot (u \nabla v) &= 0 \quad \text{in } \Omega \times (0, T), \\
\partial_t v - \Delta v + v &= u + f v 1_{\Omega_c} \quad \text{in } \Omega \times (0, T), \\
\partial_n u = \partial_n v &= 0 \quad \text{on } \partial \Omega \times (0, T), \\
u(0, \cdot) = u_0 &\geq 0, \quad v(0, \cdot) = v_0 \geq 0 \quad \text{in } \Omega,
\end{cases}
\end{aligned}
\]

being \( f : Q_c := (0, T) \times \Omega_c \rightarrow \mathbb{R} \) (the control) with \( \Omega_c \subset \Omega \subset \mathbb{R}^n \) \((n = 2, 3)\) the control domain, and the state \( u, v : Q := (0, T) \times \Omega_c \rightarrow \mathbb{R}_+ \) the cellular density and chemical concentration, respectively. Here, \( n \) is the outward unit normal vector to \( \partial \Omega \).

The existence and uniqueness of global in time weak solution \((u, v)\) for the uncontrolled system is known (see for instance [1, 4]).

In this work we study an optimal control problem subject to a chemo-repulsion system with linear production term, and in which a bilinear control
acts injecting or extracting chemical substance on a subdomain of control $\Omega_c \subset \Omega$. Existence of weak solutions are stablished (in the 3D case by using a regularity criterion), and, as a consequence, a global optimal solution together with first-order optimality conditions for local optimal solutions are deduced.

The results presented in this talk are based on [2, 3].

References


