

Minimum supports of eigenfunctions of graphs

Alexandr Valyuzhenich

Sobolev Institute of Mathematics

graphkipер@mail.ru

Let $G = (V, E)$ be a graph with the adjacency matrix $A(G)$. The set of neighbors of a vertex x is denoted by $N(x)$. Let λ be an eigenvalue of the matrix $A(G)$. A function $f : V \rightarrow \mathbb{R}$ is called a λ -eigenfunction of G if $f \neq 0$ and the equality

$$\lambda \cdot f(x) = \sum_{y \in N(x)} f(y)$$

holds for any vertex $x \in V$. In this talk we focus on the following extremal problem for eigenfunctions of graphs.

Problem 1 (MS-problem). *Let G be a graph and let λ be an eigenvalue of G . Find the minimum cardinality of the support of a λ -eigenfunction of G .*

MS-problem was first formulated by Krotov and Vorob'ev [11] in 2014 (they considered MS-problem for the Hamming graph). During the last six years, MS-problem has been actively studied for various families of distance-regular graphs [1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12] and Cayley graphs on the symmetric group [3]. In particular, MS-problem is completely solved for all eigenvalues of the Hamming graph [9, 10] and asymptotically solved for all eigenvalues of the Johnson graph [12]. In this talk we will discuss several new results on MS-problem.

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