Kirkman triple systems with many symmetries

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A *Steiner triple system* of order v, STS(v) for short, is a set V with v *points* together with a collection $\mathcal{B}$ of 3-subsets of V (*blocks*) such that that any pair of distinct points is contained in exactly one block. A *Kirkman triple system* of order v, KTS(v) for short, is a STS(v) whose blocks are partitioned in *parallel classes* each of which is a partition of the point-set V.

Steiner and Kirkman triple systems are among the most popular objects in combinatorics and their existence has been established a long time ago. Yet, very little is known about the automorphism groups of KTSs of an arbitrary order. In particular, from the very beginning of my research in design theory, I found surprising that there was no known pair (r, n) for which whenever $v \equiv r \pmod{n}$ one may claim that there exists a KTS(v) with a number of automorphisms at least close to v.

After pursuing the target of getting such a pair (r, n) for more than twenty years, we recently managed to find the following: $(39, 72)$ and $(4^e 48+3, 4^e 96)$ for any $e \geq 0$. Indeed, for $v \equiv r \pmod{n}$ with (r, n) as above, we are able to exhibit, concretely, a KTS(v) with an automorphism group of order at least equal to $v - 3$. The proof is very long and elaborated; so I will try to speak about the main ideas which led us to this result as for instance the invention of some variants of well known combinatorial objects which could be also used in the search of other combinatorial designs.