Spectral geometry of quantum graphs via surgery principles

James Kennedy
University of Lisbon
j.bernard.kennedy@gmail.com

Gregory Berkolaiko
Texas A&M University
gregory.berkolaiko@math.tamu.edu

Pavel Kurasov
Stockholm University
kurasov@math.su.se

Delio Mugnolo
University of Hagen
delio.mugnolo@fernuni-hagen.de

“Surgery” on a (metric) graph means making a small, generally local, change to its structure: for example, joining two vertices, lengthening an edge, or maybe removing an edge and reinserting it somewhere else.

We will introduce a number of sharp new surgery principles which allow one to control the eigenvalues of the Laplacian on a metric graph with any of the usual vertex conditions (natural, Dirichlet or delta). We will illustrate how these principles can be used to give new proofs and sharper versions of existing “isoperimetric”-type eigenvalue estimates by sketching a result which interpolates between the theorems of Nicaise and Band–Lévy for the first non-trivial eigenvalue of the Laplacian with natural vertex conditions.

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