Circularly compatible ones, $D$-circularity, and proper circular-arc bigraphs

Martín Safe

Departamento de Matemática, Universidad Nacional del Sur
msafe@uns.edu.ar

In 1969, Alan Tucker characterized proper circular-arc graphs as those graphs whose augmented adjacency matrices have the circularly compatible ones property. Moreover, he also found a polynomial-time algorithm for deciding whether any given augmented adjacency matrix has the circularly compatible ones property. These results allowed him to devise the first polynomial-time recognition algorithm for proper circular-arc graphs. However, as Tucker himself remarks, he did not solve the problems of finding a structure theorem and an efficient recognition algorithm for the circularly compatible ones property in arbitrary matrices (i.e., not restricted to augmented adjacency matrices only). In this work, we solve these problems. More precisely, we give a minimal forbidden submatrix characterization for the circularly compatible ones property in arbitrary matrices and a linear-time recognition algorithm for the same property. We derive these results from analogous ones for the related $D$-circular property. Interestingly, these results lead to a minimal forbidden induced subgraph characterization and a linear-time recognition algorithm for proper circular-arc bigraphs, solving a problem first posed by Basu, Das, Ghosh, and Sen [J. Graph Theory, 73(4):361–376, 2013]. Our findings generalize some known results about $D$-interval hypergraphs and proper interval bigraphs.