The fast p-Laplacian evolution equation. Global Harnack principle and fine asymptotic behaviour

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We study fine global properties of nonnegative solutions to the Cauchy Problem for the fast p-Laplacian evolution equation on the whole Euclidean space, in the so-called "good fast diffusion range". It is well-known that non-negative solutions behave for large times as B, the Barenblatt (or fundamental) solution, which has an explicit expression. We prove the so-called Global Harnack Principle (GHP), that is, precise global pointwise upper and lower estimates of nonnegative solutions in terms of B. This can be considered the nonlinear counterpart of the celebrated Gaussian estimates for the linear heat equation. To the best of our knowledge, analogous issues for the linear heat equation, do not possess such clear answers, only partial results are known. Also, we characterize the maximal (hence optimal) class of initial data such that the GHP holds, by means of an integral tail condition, easy to check. Finally, we derive sharp global quantitative upper bounds of the modulus of the gradient of the solution, and, when data are radially decreasing, we show uniform convergence in relative error for the gradients.