Lagrange multipliers and nonconstant gradient constrained problem

Sofia Giuffrè

Mediterranea University of Reggio Calabria

sofia.giuffre@unirc.it

The talk is aimed at studying a gradient constrained problem associated to a linear operator. This classical problem was subject to an intense study a few decades ago (see [1, 2, 4, 6, 8], but some very important issues were left open. In particular, we are able to prove two kinds of results (see [5]): first, we prove the equivalence of a non-constant gradient constraint problem to a suitable obstacle problem, where the obstacles solve a Hamilton-Jacobi equation in the viscosity sense (see [7]) and, second, we obtain the existence of Lagrange multipliers associated to the problem. The Lagrange multipliers exist as a Radon measure in the case that the free term of the equation $f \in L^p$, $p > 1$, whereas, if f is a positive constant, it is possible to regularize the result, namely to prove that they belong to L^2. These results have been obtained, using a new theory of infinite dimensional duality contained in [3]. The classical strong duality theory does not work in an infinite dimensional setting, when the interior of the ordering cone of the sign constraints is empty and this new theory overcomes this difficulty.

References

[1] H. Brezis, G. Stampacchia, Sur la régularité de la solution d'inéquations elliptiques, Bull. Soc. Math. France 96 (1968) 153–180.

[2] G. Cimatti, The plane stress problem of Ghizetti in elastoplasticity, Appl. Math. Optim.3 (1), 15-26 (1976).

[3] P. Daniele, S. Giuffrè, A. Maugeri, F.Raciti, Duality theory and applications to unilateral problems, J. Optim. Theory Appl. (2014) 162:718-734.

[4] L. Evans, A second order elliptic equation with gradient constraint, Comm.Part. Diff. Eq., 4 (1979), 555-572.

[5] S. Giuffrè, Lagrange multipliers and non-constant gradient constrained problem, Journal of Differential Equations 269 (1), 542-562, 2020.

[6] H. Ishii, S. Koike, Boundary regularity and uniqueness for an elliptic equation with gradient constraint, Comm. in Partial Diff. Eq., 8(4) (1983), 317- 346.

[7] P.L. Lions, Generalized Solutions for Hamilton-Jacobi Equations, Pitman Advanced Publishing Program, Boston-London-Melbourne, 1982.

[8] M. Wiegner, The $C^{1,1}$-character of solutions of second order elliptic equations with gradient constraint, Comm.Part. Diff. Eq., 6, 361-371 (1981).