On Efficient Domination for H-free bipartite graphs

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A vertex set D in a finite undirected graph G is an *efficient dominating set* (*e.d.s.* for short) of G if every vertex of G is dominated by exactly one vertex of D. The *Efficient Domination* (*ED*) problem, asks for the existence of an e.d.s. in G; it is the Exact Cover problem for the closed neighborhood hypergraph of G. ED is known to be NP-complete even for very restricted H-free graph classes such as for $2P_3$-free chordal graphs (and thus, for P_7-free graphs) while it is solvable in polynomial time for P_6-free graphs. For H-free graphs, ED is either NP-complete or polynomial, i.e., a dichotomy. However, for H-free bipartite graphs, there is no such dichotomy.

Lu and Tang showed that ED is NP-complete for chordal bipartite graphs and for planar bipartite graphs; actually, ED is NP-complete even for planar bipartite graphs with vertex degree at most 3 and girth at least g for every fixed g. Thus, ED is NP-complete for $K_{1,4}$-free bipartite graphs and for C_4-free bipartite graphs. For classes of bounded clique-width, ED is solvable in polynomial time. Dabrowski and Paulusma published a dichotomy for clique-width of H-free bipartite graphs. For instance, clique-width of $S_{1,2,3}$-free bipartite graphs is bounded.

In Discrete Applied Math. 270 (2019), we published a manuscript "On efficient domination for some classes of H-free bipartite graphs". We showed that (weighted) ED can be solved in polynomial time for H-free bipartite graphs when H is P_7 or ℓP_4 for fixed ℓ, and similarly for P_9-free bipartite graphs with vertex degree at most 3, and when H is $S_{2,2,4}$. In this talk, we also mention a polynomial time solution for P_8-free bipartite graphs (which was an open problem in our publication).