On automorphisms of direct products of abelian Cayley graphs

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The direct product of two graphs $X$ and $Y$ is denoted $X \times Y$. (It is also known as the “tensor product” or “categorical product” or “Kronecker product” or “conjunction” of $X$ and $Y$.) This is a natural construction, so any isomorphism from $X$ to $X'$ can be combined with any isomorphism from $Y$ to $Y'$ to obtain an isomorphism from $X \times Y$ to $X' \times Y'$. Therefore, the automorphism group $\text{Aut}(X \times Y)$ contains a copy of $(\text{Aut } X) \times (\text{Aut } Y)$. It is not known when this inclusion is an equality, even for the special case where $Y = K_2$ is the complete graph with only 2 vertices. (The direct product $X \times K_2$ is also known as the “canonical bipartite double cover” of $X$. The graph $X$ is said to be “stable” if equality holds in this special case.)

When $X$ is a circulant graph with an odd number of vertices (and $Y = K_2$), recent work of B. Fernandez and A. Hujdurović shows that equality holds if and only if $X$ is connected and no two vertices of $X$ have exactly the same neighbours. We will present a short, elementary proof that generalizes this theorem to the case where $X$ is a Cayley graph on an abelian group of odd order.