Subbundles of the Hodge bundle, fibred surfaces and the Coleman-Oort conjecture

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The Hodge bundle of a semistable family of complex projective curves has two nested subbundles: the flat unitary subbundle (spanned by flat sections with respect to the Gauss-Manin connection), and the kernel of the Higgs field. The latter contains the flat subbundle, but it was not clear how strict the inclusion could be. In this talk I will present some techniques to estimate the ranks of both subbundles and to construct families where they are arbitrarily different. I will also discuss some implications of this result for the classification of fibred surfaces, as well as some connections with the Coleman-Oort conjecture on the (non-)existence of totally geodesic subvarieties in the Torelli locus of principally polarized abelian varieties. This is joint work with Sara Torelli.