Should I stay or should I go? Zero-size jumps in random walks for Lévy flights

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Motivated by the fact that, in the literature dedicated to random walks for anomalous diffusion, it is disregarded if the walker does not move in the majority of the iterations because the most frequent jump-size is zero (i.e., the jump-size distribution is unimodal with mode located in zero) or, in opposition, if the walker always moves because the jumps with zero-size never occur (i.e., the jump-size distribution is bi-modal and equal to zero in zero), we provide an example in which indeed the shape of the jump-distribution plays a role. In particular, we show that the convergence of Markovian continuous-time random walk (CTRW) models for Lévy flights to a density function that solves the fractional diffusion equation is not guaranteed when the jumps follow a bi-modal power-law distribution equal to zero in zero, but, as a matter of fact, the resulting diffusive process converges to a density function that solves a double-order fractional diffusion equation. Within this framework, self-similarity is lost. The consequence of this loss of self-similarity is the emergence of a time-scale for realizing the large-time limit. Such time-scale results to span from zero to infinity accordingly to the power-law displayed by the tails of the walker's density function. Hence, the large-time limit could not be reached in real systems. The significance of this result is two-fold: i) with regard to the probabilistic derivation of the fractional diffusion equation and also ii) with regard to recurrence and the related concept of site fidelity in the framework of Lévy-like motion for wild animals.