Computing Weighted Subset Transversals in H-free Graph

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In graph transversal problems, one seeks, given a graph, to find a small subset of the vertex set — a *transversal* — that intersects every subgraph of a specified kind. Examples include vertex cover, feedback vertex set and odd cycle transversal. In *subset* versions of the problem, a subset of the vertex set is also given and the transversal that is found must also intersect this subset. In *weighted* versions, a vertex weighting is given and the aim is to find a transversal such that the sum of the weights of the transversal vertices is small.

These problems are NP-hard in general. We discuss recent work on graph transversal problems for hereditary graph classes. We will focus in particular on the problems Weighted Subset Odd Cycle Transversal and Weighted Subset Feedback Vertex Set for graph classes that are H-free. We present new algorithms that lead us to classify the complexity of the problems except in the cases where H is $2P_1 + P_3$, $P_1 + P_4$ or $2P_1 + P_4$. We note that in the latter two cases the complexity remains open even for the unweighted non-subset versions of the problem. One interesting aspect of our classification is the discovery that the complexities of weighted and unweighted versions of Subset Odd Cycle Transversal do not coincide for H-free graphs.