Discrete Fuglede conjecture on cyclic groups

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Fuglede in 1974 conjectured that a bounded domain $S \subset \mathbb{R}^d$ tiles the $d$-dimensional Euclidean space if and only if the set of functions in $L^2(S)$ admits an orthogonal basis of exponential functions.

In my talk we focus on the discrete version of Fuglede’s conjecture that can be formulated as follows. A subset $S$ of a finite abelian group $G$ tiles $G$ if and only if the character table of $G$ has a submatrix, whose rows are indexed by the elements of $S$, which is a complex Hadamard matrix. Fuglede’s original conjecture were disproved first by Tao and the proof is based on a counterexample on elementary abelian $p$-groups.

On the other hand, it is still an open question whether the discrete Fuglede’s conjecture is true on cyclic groups. In my talk I will summarize the known results concerning this question. In particular, I will present our recent result which shows that the conjecture holds on cyclic groups whose order is the product of at most 4 (not necessarily different) primes. I will introduce a geometric technique that we called ‘cube-rule’ and which is an essential tool of the proof.