How does the chromatic number of a random graph vary?

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If we pick an $n$-vertex graph uniformly at random, how much does its chromatic number vary? In 1987 Shamir and Spencer proved that it is contained in some sequence of intervals of length about $n^{1/2}$. Alon improved this slightly to $n^{1/2}/\log n$. Until recently, however, no non-trivial lower bounds on the fluctuations of the chromatic number of a random graph were known, even though the question was raised by Bollobás many years ago. We will present the main ideas needed to prove that, at least for infinitely many values $n$, the chromatic number of a uniform $n$-vertex graph is not concentrated on fewer than $n^{1/2-o(1)}$ consecutive values.

We will also discuss the Zigzag Conjecture, made recently by Bollobás, Heckel, Morris, Panagiotou, Riordan and Smith: this proposes that the correct concentration interval length 'zigzags' between $n^{1/4+o(1)}$ and $n^{1/2+o(1)}$, depending on $n$.

Joint work with Oliver Riordan.