Characterization of Sobolev functions with zero traces via the distance function from the boundary

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Let Ω be a regular domain in the Euclidean space $\mathbb{R}^n$ and let $d$ be the distance function from the boundary of Ω. A classical result of late 1980’s states that for $p \in (1, \infty)$ and $m \in \mathbb{N}$, $u$ belongs to the Sobolev space $W_0^{m,p}(\Omega)$ if and only if $u/d^m \in L^p(\Omega)$ and $|\nabla^m u| \in L^p(\Omega)$. During the consequent decades, several authors have spent considerable effort in order to relax the characterizing condition concerning requirements on the regularity of the function $u/d^m$. We present a new such condition in terms of Lorentz spaces.