Metric measure spaces and synthetic Ricci bounds

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Metric measure spaces with synthetic Ricci bounds have attracted great interest in recent years, accompanied by spectacular breakthroughs and deep new insights. In this talk, I will provide a brief introduction to the concept of lower Ricci bounds as introduced by Lott-Villani and myself, and illustrate some of its geometric, analytic and probabilistic consequences, among them Li-Yau estimates, coupling properties for Brownian motions, sharp functional and isoperimetric inequalities, rigidity results, and structural properties like rectifiability and rectifiability of the boundary. In particular, I will explain its crucial interplay with the heat flow and its link to the curvature-dimension condition formulated in functional-analytic terms by Bakry-Émery. This equivalence between the Lagrangian and the Eulerian approach then will be further explored in various recent research directions: i) distribution-valued Ricci bounds which e.g. allow singular effects of non-convex boundaries to be taken into account, ii) time-dependent Ricci bounds which provide a link to (super-) Ricci flows for singular spaces, iii) upper curvature bounds.