

Spanning bipartite subgraphs of triangulations of a surface

Kenta Noguchi

Tokyo University of Science

noguchi@rs.tus.ac.jp

A triangulation (resp., a quadrangulation) of a surface S is an embedded graph (possibly with multiple edges and loops) on S with each face bounded by a closed walk of length 3 (resp., 4). This talk focuses on the relationship between triangulations and quadrangulations of a surface.

(a) Extension of a graph G is the construction of a new graph by adding edges to some pairs of vertices in G . Obviously, every quadrangulation G of any surface can be extended to a triangulation by adding a diagonal to each face of G . If we require some properties for the resulting triangulation, the problem might be difficult and interesting. We prove that every quadrangulation of any surface can be extended to an Eulerian triangulation. Furthermore, we give the explicit formula for the number of distinct Eulerian triangulations extended from a given quadrangulation of a surface. These completely solves the problem raised by Zhang and He [5].

(b) It is easy to see that every loopless triangulation G of any surface has a quadrangulation as a spanning subgraph of G . As well as (a), if we require some properties for the resulting quadrangulation, the problem might be difficult and interesting. Kündgen and Thomassen [1] proved that every loopless Eulerian triangulation G of the torus has a spanning nonbipartite quadrangulation, and that if G has sufficiently large face width, then G also has a bipartite one. We prove that a loopless Eulerian triangulation G of the torus has a spanning bipartite quadrangulation if and only if G does not have K_7 as a subgraph.

This talk is based on the papers [2, 3, 4].

References

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