Dynamics of visco-elastic bodies with a cohesive interface

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We consider the dynamics of elastic materials with a common cohesive interface (or a domain with a prescribed cohesive fracture). In the bulk, the evolution is provided by linearized elasto-dynamics with Kelvin-Voigt visco-elastic dissipation, while on the interface the evolution is governed by a system of Karush-Kuhn-Tucker depending on the crack opening and on the internal variable. The weak formulation reads

\[
\begin{aligned}
\rho \ddot{u}(t) + \partial_v \mathcal{E}(u(t)) - \langle F(t), u \rangle + \partial_{[u]} \Psi(\xi(t), [u(t)]) + \partial_\dot{v} \mathcal{R}(\dot{u}) &\geq 0, \\
\dot{\xi}(t)(\xi(t) - [u(t)]) = 0 \quad \text{and} \quad [u(t)] \leq \xi(t), \\
u(0) = u_0, \quad \dot{u}(0) = u_1,
\end{aligned}
\]

where \( \mathcal{E} \) is the elastic energy, \( F \) is the external force, \( \mathcal{R} \) is Kelvin-Voigt visco-elastic dissipation, while \( \Psi \) is the interface cohesive potential, concave under loading and quadratic under unloading.

First we provide existence of a time-discrete evolution by means of incremental minimization problems (fully implicit in the displacement) and then its time-continuous limit, which satisfies the energy identity

\[
\mathcal{E}(u(t)) + \Psi([u(t)], \xi(t)) + \mathcal{K}(\dot{u}(t)) = \mathcal{E}(u_0) + \Psi([u_0], \xi_0) + \mathcal{K}(v_0) + \\
+ \int_0^t \langle F(s), \dot{u}(s) \rangle \, ds - \int_0^t \partial_v \mathcal{R}(\dot{u}(s))[\dot{u}(s)] \, ds.
\]

Finally, we discuss the strong formulation of the system, with acceleration in \( L^2 \) and equilibrium of forces on the interface.