On coverings and perfect colorings of hypergraphs

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In this talk, we consider perfect colorings (also known as equitable partitions) of hypergraphs. A perfect $k$-coloring of a hypergraph is a coloring of its vertices into $k$ colors such that colors of hyperedges incident to a vertex is defined by its color. A transversal of a hypergraph is one of the simplest examples of perfect 2-colorings. While perfect colorings of graphs are well known and extensively applied, similar objects for hypergraphs are hardly studied. The main aim of the talk is to show that perfect colorings of hypergraphs have most of the nice algebraic properties of colorings of graphs.

Firstly, we define the multidimensional parameter matrix of a perfect coloring of a hypergraph, study its eigenvalues, and connect them with the parameters of the corresponding perfect coloring of the bipartite representation. Next, we introduce coverings of hypergraphs and show that for a vast class of hypergraphs there exist coverings that can be partitioned into perfect matchings. At last, we establish that if two hypergraphs have the same minimal perfect coloring then there is a hypergraph covering both of them.