L^p estimates for wave equations with specific Lipschitz coefficients

Pierre Portal
Australian National University
pierre.portal@anu.edu.au

Dorothee Frey
Karlsruhe Institute of Technology
dorothee.frey@kit.edu

For the standard linear wave equation $\partial_t^2 u = \Delta u$, the solution at time t belongs to $L^p(\mathbb{R}^d)$ for initial data $u(0,.) \in W^{(d-1)|\frac{1}{p}-\frac{1}{2}|,p}$, $\partial_t u(0,.) = 0$. This is a classical result of Peral/Myachi from the 1980's, which motivated the development of Fourier Integral Operator theory. It is optimal in terms of the order of the Sobolev space of initial data. In this talk, we discuss an extension of this result for certain wave equations, such as $\partial_t^2 u = \sum_{j=1}^{d} \partial_j a_j \partial_j u$, where a_j are $C^{0,1}$ functions bounded from above and below and depending only on the j-th variable. The fact that such a result holds is somewhat surprising, given that other space-time (Strichartz) estimates typically fail for coefficients rougher than $C^{1,1}$. The proof is based on an approach to FIO theory via phase space Hardy spaces (recently developed by Hassell, P., Rozendaal) combined with operator theoretic and harmonic analytic methods. The talk presents the scheme of the proof, focusing on the ideas behind each of the key steps.