# Structure-Preserving Interpolation for Bilinear Systems

Steffen W. R. Werner
*Max Planck Institute for Dynamics of Complex Technical Systems*
werner@mpi-magdeburg.mpg.de
Peter Benner
*Max Planck Institute for Dynamics of Complex Technical Systems*
benner@mpi-magdeburg.mpg.de
Serkan Gugercin
*Virginia Polytechnic Institute and State University*
gugercin@vt.edu

The modeling of natural processes as population growth, mechanical structures and fluid dynamics, or stochastic modeling often results in bilinear time-invariant dynamical systems

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + \sum_{j=1}^{k} N_j x(t) u_j(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

with $E, A, N_j \in \mathbb{R}^{n \times n}$, for $j = 1, \ldots, m$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$. The aim of model reduction for (1) is the reduction of related computational resources, like time and memory for the simulation of (1), by the reduction of internal states $n$, while approximating the input-to-output behavior of the system. Often related to the underlying applications, bilinear systems (1) can have special structures that one wants to preserve in the reduced-order model as, e.g., in case of bilinear mechanical systems

$$\begin{aligned} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) &= \sum_{j=1}^{m} N_{\mathrm{p},j} q(t) u_j(t) + \sum_{j=1}^{m} N_{\mathrm{v},j} \dot{q}(t) u_j(t) + B_{\mathrm{u}} u(t), \\ y(t) &= C_{\mathrm{p}} q(t) + C_{\mathrm{v}} \dot{q}(t), \end{aligned} \tag{2}$$

with $M, D, K, N_{\mathrm{p},j}, N_{\mathrm{v},j} \in \mathbb{R}^{n \times n}$, for $j = 1, \ldots, m$, $B_{\mathrm{u}} \in \mathbb{R}^{n \times m}$ and $C_{\mathrm{p}}, C_{\mathrm{v}} \in \mathrm{R}^{p \times n}$.

In case of linear systems, structured-preserving interpolation of the underlying transfer function in the frequency domain can be used to efficiently construct reduced-order models with the same structure as the original system [1].

We present an extension of the structure-preserving interpolation framework to the bilinear system case, which we describe in the frequency domain by general multivariate transfer functions

$$G_k(s_1, \ldots, s_k) = \mathcal{C}(s_k)\mathcal{K}(s_k)^{-1} \left( \prod_{j=1}^{k-1} (I_{m^{j-1}} \otimes \mathcal{N}(s_{k-j}))(I_{m^j} \otimes \mathcal{K}(s_{k-j})^{-1}) \right) \times (I_{m^{k-1}} \otimes \mathcal{B}(s_1)), \tag{3}$$

for $k \geq 1$ and where $\mathcal{N}(s) = \begin{bmatrix} \mathcal{N}_1(s) & \ldots & \mathcal{N}_m(s) \end{bmatrix}$ with the matrix functions $\mathcal{C} : \mathbb{C} \to \mathbb{C}^{p \times n}$, $\mathcal{K} : \mathbb{C} \to \mathbb{C}^{n \times n}$, $\mathcal{B} : \mathbb{C} \to \mathbb{C}^{n \times m}$, $\mathcal{N}_j : \mathbb{C} \to \mathbb{C}^{n \times n}$ for $j = 1, \ldots, m$. We develop a projection-based, structure-preserving interpolation framework for bilinear systems associated with (3) that allows the efficient construction of reduced-order structured bilinear systems.

# References

[1] C. A. Beattie and S. Gugercin. Interpolatory projection methods for structure-preserving model reduction. *Syst. Control Lett.*, 58(3):225–232, 2009. doi:10.1016/j.sysconle.2008.10.016.