Critical semilinear fractional elliptic problems involving an inverse fractional operator

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In this talk we will study the existence of positive solutions for a problem related to a higher order fractional differential equation involving a nonlinear term depending on a fractional differential operator,

\[
\begin{align*}
(-\Delta)^\alpha u &= \lambda u + (-\Delta)^\beta |u|^{p-1}u \quad \text{in } \Omega, \\
(-\Delta)^j u &= 0 \quad \text{on } \partial \Omega, \quad \text{for } j \in \mathbb{Z}, \quad 0 \leq j < [\alpha],
\end{align*}
\]

where \( \Omega \) is a bounded domain in \( \mathbb{R}^N \), \( 0 < \beta < 1 \), \( \beta < \alpha < \beta + 1 \) and \( \lambda > 0 \).

In particular, we will show study the following fractional elliptic problem,

\[
\begin{align*}
(-\Delta)^{\alpha-\beta} u &= \lambda (-\Delta)^{-\beta} u + |u|^{p-1}u \quad \text{in } \Omega, \\
u &= 0 \quad \text{on } \partial \Omega,
\end{align*}
\]

proving existence or nonexistence of positive solutions depending on the parameter \( \lambda > 0 \), up to the critical value of the exponent \( p \), i.e., for \( 1 < p \leq 2^*_\mu - 1 \) where \( \mu := \alpha - \beta \) and \( 2^*_\mu = \frac{2N}{N-2\mu} \) is the critical exponent of the Sobolev embedding.

The results are mainly collected in the following paper,