

Treewidth versus clique number: a complete dichotomy for one forbidden structureKenny Štorgel*Faculty of Information studies in Novo mesto*`kennystorgel.research@gmail.com`

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Treewidth is an important graph invariant, relevant for both structural and algorithmic reasons. A necessary condition for a graph class to have bounded treewidth is the absence of large cliques. We study graph classes closed under taking induced subgraphs in which this condition is also sufficient, which we call (tw, ω) -bounded. For six graph containment relations (the subgraph, topological minor, and minor relations, as well as their induced variants) we give a complete characterization of graphs H for which the class of graphs excluding H is (tw, ω) -bounded.

Our results yield an infinite family of χ -bounded induced-minor-closed graph classes and imply that the class of 1-perfectly orientable graphs is (tw, ω) -bounded, leading to linear-time algorithms for k -coloring 1-perfectly orientable graphs for every fixed k . This answers a question of Brešar, Hartinger, Kos, and Milanič (2018) and one of Beisegel, Chudnovsky, Gurvich, Milanič, and Servatius (2019), respectively. We also reveal some further algorithmic implications of (tw, ω) -boundedness related to list k -coloring and clique problems.