On the variance of the nodal volume of arithmetic random waves

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We discuss arithmetic random waves on the $d$-dimensional torus $\mathbb{R}^d/\mathbb{Z}^d$. Their zero set and its volume are related to the study of linear correlations of lattice points on the sphere of radius $\sqrt{n}$ in $\mathbb{R}^d$ ($n \geq 2$). In this talk we look for bounds on the variance of the nodal volume. The problem has been solved (in the stronger form of an asymptotic with power saving) in dimension $d = 2, 3$ by using a combination of number theory and graph theory. In this seminar we will explain what is known in dimension $d \geq 4$. As the dimension increases, analysis gives the best results. The main input is a result that follows from the proof of the $l^2$-decoupling conjecture by Bourgain and Demeter.