Multiple entire solutions to the curl-curl problem with critical exponent

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We prove the existence of infinitely many solutions with diverging energy to the problem
\[ \nabla \times \nabla \times U = |U|^4 U \quad \text{in} \quad \mathbb{R}^3. \]

We consider vector fields of the form
\[ U(x) = \frac{u(x)}{r} \left( \frac{-x_2}{x_1}, 0 \right) \]
with \( r = \sqrt{x_1^2 + x_2^2} \) and \( u(x) = u(r, x_3) \) to reduce the curl-curl operator to the vector Laplacian; at the same time we consider an isometric isomorphism between \( D^{1,2}(\mathbb{R}^3, \mathbb{R}^3) \) and \( H^1(S^3, \mathbb{R}^3) \) to recover compactness.