Numerical solution of Optimal Transport Problem on graphs

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The Dynamical Monge-Kantorovich (DMK) model is dynamical system of equations whose steady state has been related to the solution of the Optimal Transport Problem with cost equal to the Euclidean distance. In this talk we present a re-adaptation into a graph setting of the DMK model, that can be rewritten in the form of a Gradient Flow. Using this formulation, we can solve the OTP on graphs with cost given by the shortest path distance looking at the long-time solution of the Gradient Flow equations. To this aim, we discretized the Gradient Flow equation via backward Euler time-stepping, in order to use larger time steps, getting faster convergence toward the optimal solution. The non-linear equations arising from such implicit time-stepping scheme are solved via Newton-Raphson Method. Thus, the optimization problem is reduced to the solution of a sequence of large and sparse saddle point linear systems, for which efficient preconditioners have to be build. In this talk we present different preconditioning approaches to tackle this problem.