Self-stabilizing processes

Jacques Levy Vehel
Case Law Analytics
jacques.levy-vehel@inria.fr

Kenneth Falconer
St Andrews University
kjf@st-andrews.ac.uk

A self-stabilizing processes \( \{Z(t), t \in [t_0, t_1]\} \) is a random process which when localized, that is scaled to a fine limit near a given \( t \in [t_0, t_1] \), has the distribution of an \( \alpha(Z(t)) \)-stable process, where \( \alpha : \mathbb{R} \to (0,2) \) is a given continuous function. Thus the stability index near \( t \) depends on the value of the process at \( t \). In the case where \( \alpha : \mathbb{R} \to (0,1) \), we first construct deterministic functions which satisfy a kind of autoregressive property involving sums over a plane point set \( \Pi \). Taking \( \Pi \) to be a Poisson point process then defines a random pure jump process, which we show has the desired localized distributions.

When \( \alpha \) may take values greater than 1, convergence of the considered sums may no longer be absolute. We generalize the construction in two stages, firstly by setting up a process based on a fixed point set but taking random signs of the summands, and then randomizing the point set to get a process with the desired local properties.