Convexity properties of the isoperimetric profile

Giorgio Saracco
Scuola Internazionale Superiore di Studi Avanzati
gsaracco@sissa.it

Gian Paolo Leonardi
Università di Trento
gianpaolo.leonardi@unitn.it

Given an open, bounded set $\Omega$ we consider the isoperimetric profile $J$ that to each volume $V \in [0, |\Omega|]$ associates the least perimeter $P(E)$ among Borel subsets $E$ of $\Omega$ needed to enclose the given volume. We shall prove that for a wide class of planar sets, which encompasses convex sets, there exists a threshold $\bar{V}$ such that $J$ is concave below it and convex above it. Moreover, $J^2$ is globally convex. In order to prove these properties, a full characterization of the isoperimetric sets will be provided. Some comments on the $n$-dimensional case will be given.