Eilenberg-Moore, Kleisli and descent factorizations

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Every functor that has a left adjoint has two well-known factorizations. The first one is through the category of Eilenberg-Moore algebras of the induced monad, while the second one is the factorization through the category of free coalgebras (co-Kelsili category) of the induced comonad. As usual in category theory, we also have the dual cases: a functor that has a right adjoint has a factorization through the category of Eilenberg-Moore coalgebras of the induced comonad, and other factorization through the Kleisli category of the induced monad.

More generally, if the functor has a codensity monad (resp. a op-codensity comonad), the functor has a factorization through the category of Eilenberg-Moore algebras (resp. the free coalgebras) (see, for instance, [3, Section 3]).

In [3], given a 2-category $\mathbb{A}$ satisfying suitable hypothesis, we show that every morphism inside a 2-category with opcomma objects (and pushouts) has a 2-dimensional cokernel diagram which, in the presence of the descent objects, induces a factorization of the morphism. We show that these factorizations generalize the usual Eilenberg-Moore and Kleisli factorizations.

The result is new even in the case $\mathbb{A} = \mathsf{Cat}$. In this case, we have that every functor has a factorization through the category of descent data of its 2-dimensional cokernel diagram. We show that, if a functor F has a left adjoint, this descent factorization coincides with the factorization through the category of algebras. Dually, if F has a right adjoint, this descent factorization coincides with the factorization through the category of coalgebras.

This specializes in a new connection between monadicity and descent theory, which can be seen as a counterpart account to the celebrated Bénabou-Roubaud Theorem (see [1] or, for instance, [2, Theorem 1.4] and [4, Section 4]). It also leads in particular to a (formal) monadicity theorem.

In this talk, we shall give a sketch of the ideas and constructions involved in this particular case of $\mathbb{A} = \mathsf{Cat}$.

References:

[1] J. Bénabou and J. Roubaud. Monades et descente. C. R. Acad. Sci. Paris Ser. A-B, 270:A96–A98, 1970.

[2] F. Lucatelli Nunes. Pseudo-Kan extensions and descent theory. Theory Appl. Categ., 33:No. 15, 390–444, 2018.

[3] F. Lucatelli Nunes. Semantic Factorization and Descent. DMUC preprints, 19-03, 2019. arXiv: 1902.01225

[4] F. Lucatelli Nunes. Descent Data and Absolute Kan Extensions. DMUC preprints, 19-19, 2019. arXiv: 1606.04999