Rough volatility: SDE driven by Hölder continuous noise and unbounded drift

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Having in mind possible stochastic volatility models in finance, we consider an SDE driven by a general Hölder continuous noise. The drift $b$ is exploding with a control from below:

\[ b(t,y) > C_T(y - \varphi(t))^{-\gamma} \quad (\gamma > 0), \]

where $\varphi$ is a continuous function and $C_T, \gamma$ are constants. We study the solution of such SDE and its properties. In particular, we prove that it has a unique solution which is bound preserving.

Furthermore, modifying the controls on the drift, we obtain an SDE sandwiched between two given bounds $\varphi$ and $\psi$, with $\psi(x) > \varphi(x)$. This sandwich solution turns out to be the most useful for applications.

Among the properties presented, we show that the solution admits all moments. This remarkable result paves the way for efficient numerical methods.