Wigner’s theorem in normed spaces

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Let \((H, \langle \cdot, \cdot \rangle)\) and \((K, \langle \cdot, \cdot \rangle)\) be inner product spaces over \(F \in \{\mathbb{R}, \mathbb{C}\}\) and suppose that \(f: H \to K\) is a mapping satisfying

\[|\langle f(x), f(y) \rangle| = |\langle x, y \rangle|, \quad x, y \in H.\] (1)

Then the famous Wigner’s unitary–antiunitary theorem says that \(f\) is a solution of (1) if and only if it is phase equivalent to a linear or an anti-linear isometry, say \(U\), that is,

\[f(x) = \sigma(x)Ux, \quad x \in H,\]

where \(\sigma: H \to F, |\sigma(x)| = 1, x \in H\), is a so called phase function. In this talk several generalizations of this theorem to the setting of normed spaces will be presented.